THE LATTICE BOLTZMANN METHOD: A NEW APPROACH TO RADIATIVE TRANSFER THROUGH CLOUDS

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1. INTRODUCTION

The radiative effects of clouds on operational sensors and their associated impacts on systems in use by the Department of Defense (DoD) are generally not well understood. In the case of most DoD operations, it is features of clouds at relatively small spatial scales that are of most importance. A typical infrared seeker on a guided munition, for example, must be able distinguish targets from a possible cloudy background at spatial scales that are equivalent to individual cumulus cloud elements. In this regime, the inhomogeneous three-dimensional nature of cloud fields is significant and traditional methods of treating clouds as homogeneous layers in radiometric calculations break down.

For the present work, we investigate the modeling of cloud radiative effects at very fine scales (e.g., tens of meters). At these spatial scales, the plane parallel assumptions often invoked by large-scale treatments of clouds are not valid and the full 3D inhomogeneity of cloud fields must be considered.

Radiative transport models that are applicable to finite clouds at cumulus scales are not widely available. Treatments of this problem typically involve the simulation of cloud radiative interactions using stochastic Monte Carlo methods or diffusion-limit approximations to the full radiative transfer equation. Although these methods are somewhat successful in certain applications, they each have certain undesirable attributes. Monte Carlo methods, for example, are inherently noisy and require a large number of photon simulations to produce useful results. To compensate for this, Monte Carlo simulations are typically run in reverse sense in that a given view geometry is prescribed and individual photons are tracked as they interact with the cloud media backward in time.

Recently, a new tool has emerged to model various transport phenomena based on a particle method using discrete cellular automata techniques. To date, the primary applications of this method are related to the field of computational fluid dynamics. These so-called lattice-gas methods simulate a complex dynamical system by constructing a microscale world in which space, time and velocity are discretized and fictitious particles interact with each other and their environment. In the macroscopic limit, these particles describe the time-dependent solution to a system of PDEs (e.g. Navier Stokes fluid flow) without constructing any finite-difference (or finite element) approximations to the PDEs themselves. Boon (1991) gives a review of lattice-gas methods for computational fluid dynamics applications.

From the field of lattice-gases, a separate, but closely related method has emerged in which the modeling of particles occurs at a scale that is intermediate between the microscale of the individual particles and the macroscale where the modeled physics is observed. In this intermediate, or mesoscale, one describes distributions of the particles rather than the particles themselves. This technique is known as the lattice Boltzmann method (cf., Succi, 1991 for a review). The advantage of modeling at the mesoscale is that the scale of the problem can be increased and the noise that is due to the underlying assumptions of molecular chaos in lattice-gas methods is eliminated. One of the disadvantages of this approach, however, is that correlations between individual particles are lost due to the fact that particles are treated as ensembles.

In the present work, we have adapted the lattice Boltzmann method to the problem of three dimensional radiative transport though inhomogeneous liquid water clouds. This is a novel approach to the problem and is potentially a powerful tool for modeling cloud radiative properties at the scales necessary for sensor simulation. It has several desirable features as compared to other particle-based methods. In particular, lattice Boltzmann methods operate in a forward-in-time mode and produce results which are independent of the view geometry. Another significant feature of the method is that the algorithms are inherently efficient on parallel computer architectures due to the local computation involved.

In Section 2 we describe the lattice Boltzmann method in detail and its application to cloud radiation. Section 3 presents some results derived from the method using idealized simple clouds as well as fully 3D complex cloud geometries. These results are compared with Monte Carlo simulations. In Section 4, we discuss the potential application and limitations of the lattice Boltzmann method.

2. METHOD

The Boltzmann equation for linear photonic transport can be developed by considering the change, dN, in time dt of the number of photons with velocity in d^3v about v which are located in a small volume V with surface S about the point r. Then,

$$dN = d^3v dt \int_V \frac{\partial \psi(\mathbf{r}, \mathbf{v}, t)}{\partial t} d^3r, \tag{1}$$

where ψ (r, v, t) is the expected number of photons in d^3r about r with velocities in d^3v about v at time t. In the time interval dt, the balance condition on dN is:

$$\begin{aligned} dN = &- \text{net number flowing out of S} \\ &- \text{number suffering collisions} \\ &+ \text{number scattered into } d3v \\ &+ \text{number produced by sources within } d3r \\ &- \text{number removed by sinks within } d^3r. \end{aligned} \tag{2}$$

A Boltzmann equation can be developed from Eqs. (1) and (2):

$$\mathbf{B}\psi(\mathbf{r}, \mathbf{v}, t) = q(\mathbf{r}, v, t), \tag{3}$$

where the Boltzmann operator, B, is defined as,

$$\mathbf{B} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + v\sigma(\mathbf{r}, \mathbf{v}) - \int v'\sigma(\mathbf{v}' \rightarrow \mathrm{v}, \mathbf{r}) dv', \tag{4}$$

where $\sigma(\mathbf{r}, \mathrm{v})$ is a scattering cross section and q(r, v) represents the net source of photons. Classical linear transport theory is aimed at finding solutions to Eq. (3) (cf. Case and Zweifel, 1967).

In the lattice Boltzmann method, we wish to construct a form of the Boltzmann equation analogous to Eq. (3), but where space and time (hence velocity) are discrete. To accomplish this, we define a set of lattice vectors, e_i, (i = 1..13), where i is a lattice direction index and B is the total number of directions (velocities) in the lattice. Then, the lattice Boltzmann equation can take the form,

$$\Delta f_i = f_i'(\mathbf{r} + \Delta\mathbf{r}, t + \mathrm{At}) - f_i(\mathbf{r}, t) = -\Omega_i(\mathbf{r}, t), \tag{5}$$

where, f_i represents the expected number of photons at a node in the lattice traveling with unit speed in the direction of e_i and Ω_i is a collision operator which acts to redistribute the photons traveling in each of the lattice directions at each lattice node. In the present context, this collision term represents the physics of the photon transport (e.g. scattering, absorption and emission).

The lattice Boltzmann method can be considered as two distinct steps–the first being the collision operation described by Eq. (5) and the second a propagation or streaming step. In streaming, the values of each of the f_i at each node are simply moved to the adjacent lattice node pointed to by the associated e_i. This step is key to the scalability of the lattice Boltzmann method because streaming happens synchronously. In other words, the information contained in the entire set of photons is moved at once everywhere in the lattice. This is in contrast to Monte Carlo methods where each photon moves independently of the others due to the independent nature of each photon.

The collision term may take any number of forms; however, one convenient form often used in fluid transport problems is a single-time relaxation towards equilibriume..g.,

$$\Omega_i = \frac{1}{\tau}(f_i - f_i^*), \tag{6}$$

where τ is a relaxation parameter and f_i^* is an equilibrium distribution function for the lattice photons. This form of the collision term is analogous to the so-called BGK approximation to the Boltzmann equation and is commonly known as the lattice-BGK equation.

The equilibrium distribution function, f_i^*, is difficult to determine in the general case of linear transport. However, in the present case, we may construct f_i^* rather simply by considering the "microphysics" of the photon interactions. For simplicity, consider a case of pure isotropic scattering (i.e. no absorption or emission). In this case, the equilibrium distribution function is equal in all lattice directions (i.e., f_i = cork). The only relevant constraint is that energy be conserved during the collision. This is equivalent to:

$$\sum_{i=1}^{B} \Omega_i = 0< \tag{7}$$

This condition implies that,

$$f_i^* = \rho/B. \tag{8}$$

It is apparent from the form of the collision operator Eq. (6) that the relaxation parameter, τ, governs the magnitude of the scattering in a collision. There are two limiting cases involving τ to note. For $\tau = 1$, Eq. (6) reduces to $f_i' = f_i^*$, where f_i' is the after-collision value of f_i. This represents a complete scattering of all of the incident photons. At the limit of $\tau = \infty$, we have $f_i' = f_i$, which can be interpreted as pure transmission (no scattering). In general, τ, is a measure of the (scattering) optical depth and is treated as a spatially varying quantity.

To determine the relationship between optical depth, δ, we consider a special case of the equilibrium distribution function-specifically,

$$f_i^* = \frac{1}{B} f_i. \tag{9}$$

In this case, we have effectively eliminated the contribution of scattered photons being re-scattered into the f_i direction. This eliminates the effects of multiple scattering and allows a direct comparison with Beer's Law,

$$I(x) = I(0)e^{-\delta x}, \tag{lo}$$

where I is radiometric intensity and x is a measure of distance.

The comparison of the lattice Boltzmann radiative transport model to Beer's law proceeds by considering the ratio f_i'/f_i for a single collision (or equivalently over a single grid space) for the equilibrium distribution given by Eq. (9). We have from Eqs.(5, 6 and 9):

$$\begin{aligned} \frac{f_i'}{f_i} &= 1 - \frac{\Omega}{f_i} \\ &= 1 - \frac{1}{\tau}\left(1 - \frac{c}{B}\right). \end{aligned} \tag{11}$$

By treating the f_is as intensities, we have by comparison to Eq.(9),

$$\delta = -ln\left[1 - \frac{1}{\tau}\left(1 - \frac{c}{B}\right)\right]. \tag{12}$$

Nonconservative and anisotropic scattering are treated in the lattice Boltzmann method via a simple generalization of the equilibrium distribution function Eq. (8):

$$f_i^* = c\frac{\sum_{j=1}^{B} \xi_{ij} f_j}{\sum_{j=1}^{B} \xi_{ij}}, \tag{13}$$

where the $\xi_i j$ represent discrete values of the scattering phase function for the angle between $\hat{\mathbf{e}}_i$ and $\hat{\mathbf{e}}_j$ and c is chosen such that,

$$c > 1 \rightarrow Emission$$
$$c = 1 \rightarrow Conservation$$
$$0 < c < 1 \rightarrow Absorption \tag{14}$$

3. RESULTS

The scattering of photons gas from a columnar beam of light entering a homogeneous gas was shown in an earlier work (Mozer and Caudill, 1995). For this case, it was shown that the results of the LBRTE method agreed well with a traditional forward Monte Carlo approach. Here we extend this demonstration to include a inhomogeneous scattering medium–specifically a simulated cumulus cloud.

A physics and fractal-based cloud model (Cianciolo, 1996) was used to generate a field of cloud liquid water content representative of a cumulus cloud. Values of liquid water content were calculated on a cubical grid with 2-km sides and 2-m spacing. For this case, the optical depth across a volume element is considered proportional to the liquid water content in that voxel.

Figure (1) shows the results of a preliminary LBRTE calculation through the synthetic cloud field. In this case, the illumination is from the top of the cloud field (zenith angle zero) and no diffuse sources are included. The figure shows the field of photons exiting the computational cube at each visible face. Due to the geometry of the problem, all of the photons emitted along the lateral faces of the cube are a result of scattering within the volume. The bottom face represents transmission of the incident flux through the cloud volume.

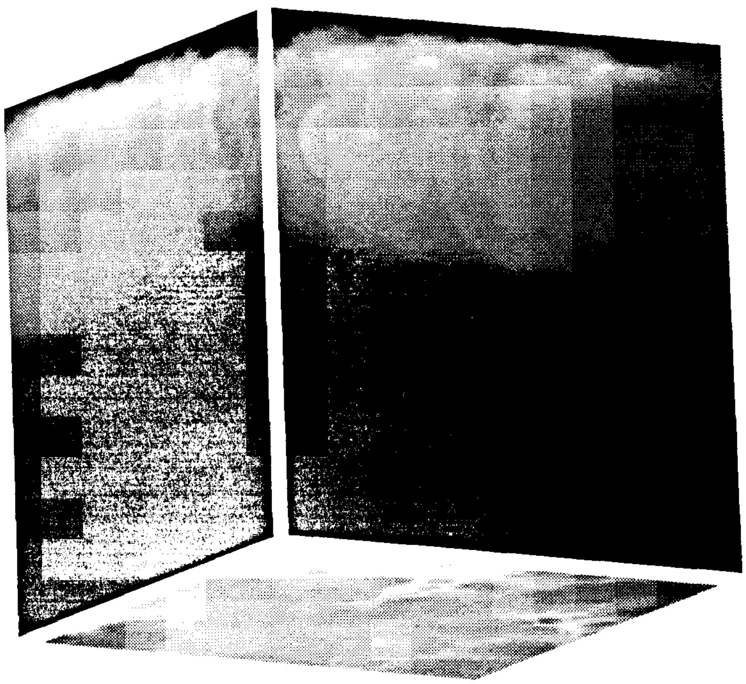

Fig. 1. Lattice Boltzmann radiative transfer calculation through a field of liquid water content representative of a cumulus cloud. The illumination is from the top. The images formed on the faces of the cube represent the photons exiting that face. No diffuse sources were included in this calculation.

3.1 Timing

One of the major benefits to the LBRTE approach is its efficiency on a parallel computer platform. We have calculated the CPU times required to perform the LBRTE calculation through the cumulus cloud shown in Fig. (1) on a parallel cluster of Pentium-based workstations. Figure (2) shows in inverse of the execution time in seconds versus the number of processors used in the calculation. Also shown in Fig. (2) are the timing results for a forward Monte Carlo (MC) code which tracked 2×10^6 individual photons.

It is apparent from the figure that the LBRTE methods are more efficient that than the MC calculation for the cases shown. However, the MC results exhibit a nearly perfect linear scalability whereas the LBRTE results depart from the straight line shown in the figure. By itself, this fact leads to the conclusion that the MC method would be more efficient when many more processors were used. However, one must consider the level of noise required in the final result. We chose to run the MC code using 2×10^6 photons because it is roughly equivalent to the number of sites in the LBRTE lattice. However, the images produced by the MC method (not shown) contain a large amount of noise. In order to suppress this noise, more photons must be run through the cloud. This leads to longer execution times. Therefore, when comparing the efficiency of the LBRTE and MC methods for a given problem, one must specify the maximum amount of noise which can be tolerated in the result.

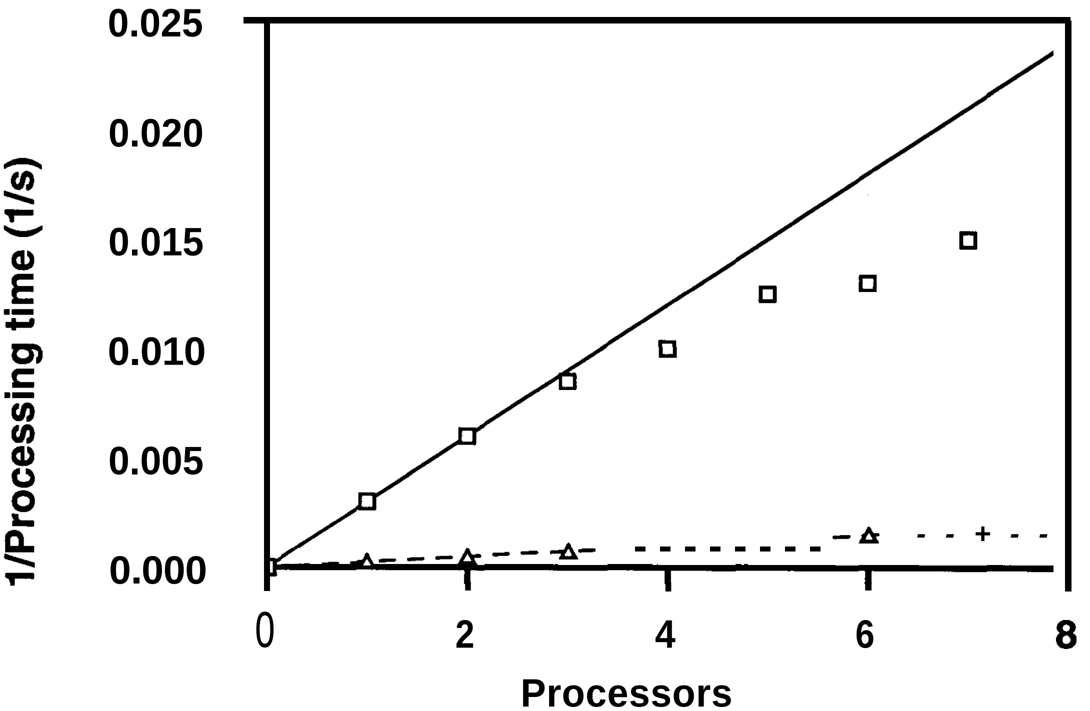

Fig. 2. Timing results for calculation shown in Fig.(1). Squares represent results for LBRTE calculation. Triangles are for a Monte Carlo calculation involving 2×10^6 photons. The solid and dashed lines represent perfect linear scaling for each method respectively

4. CONCLUSION

The Lattice Boltzmann Radiative Transfer Equation (LBRTE) method seems to be an efficient and effective method to calculate the transport of monochromatic radiation through a non-homogeneous medium. Because the LBRTE method performs an explicit streaming of photons in a discrete lattice, it is particularly adept at solving problems where multiple scattering is significant. The LBRTE method is capable of producing accurate quantitative results for cases of isotropic scattering as well as for a Rayleigh phase function. Problems involving a highly peaked scattering phase require many lattice directions to resolve the angular streaming which reduces the efficiency of the LBRTE method, Although this is a severe limitation to the method, it is expected that the LBRTE scheme will be useful for some problems–such as the scattering of microwave radiation by atmospheric clouds.

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